Trustworthiness in Robust Hypersonic Trajectory Planning

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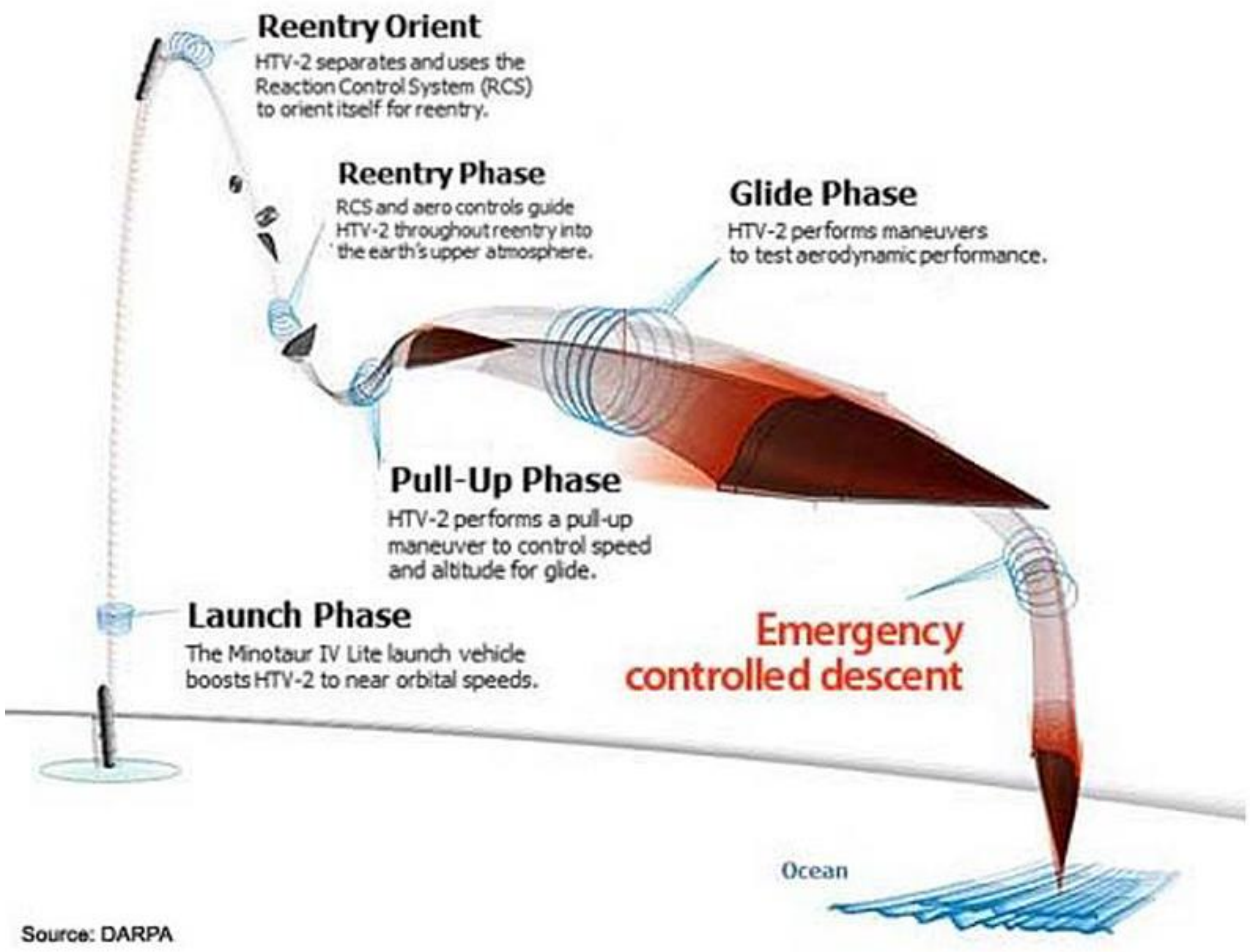
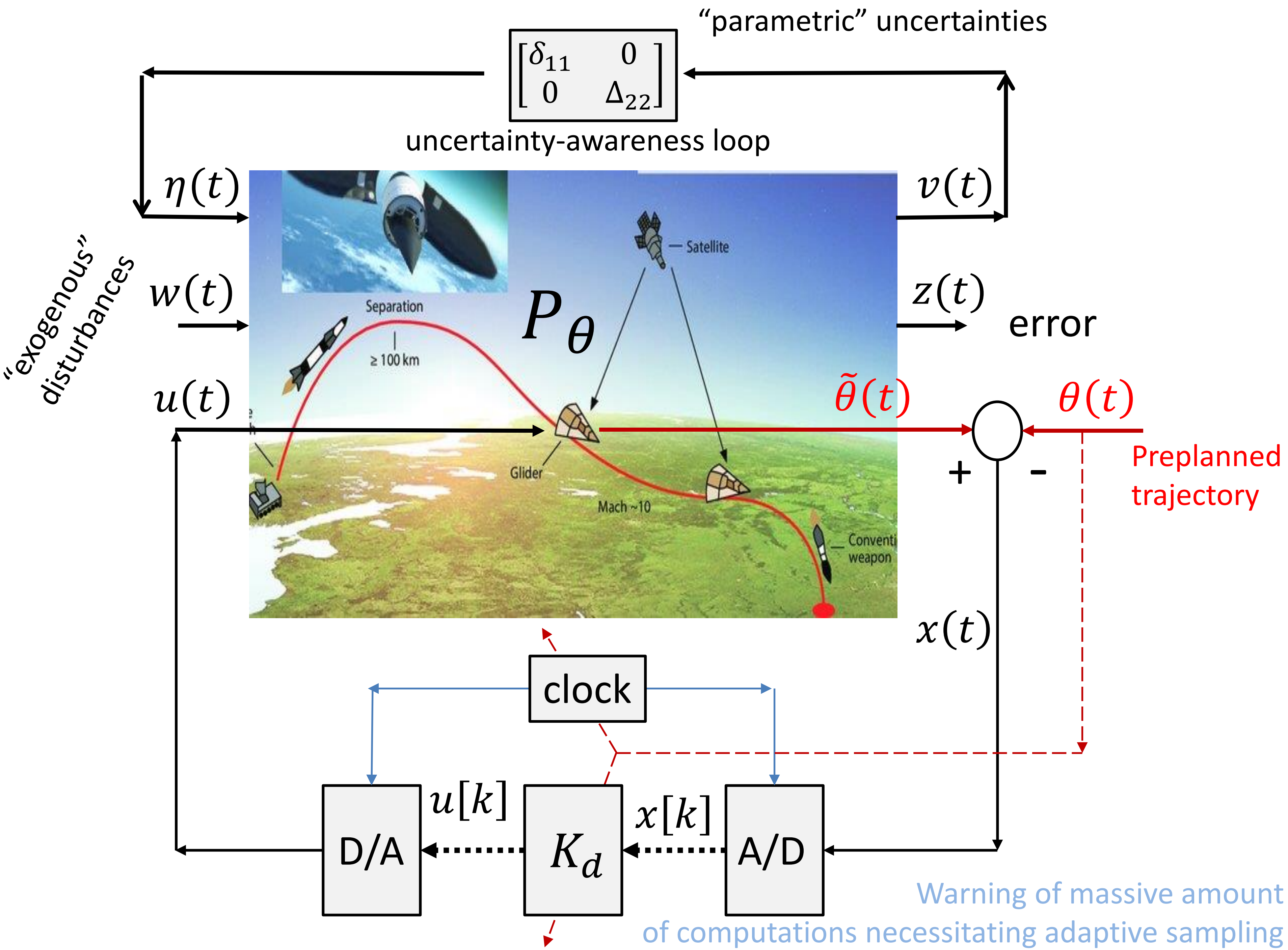
*"This solicitation is seeking advancement **to build trust**… and metrics and to quantify uncertainty versus performance… Proposed solutions should.. focus.. on **ways to build trust and confidence** in mission planning. New **methods to improve robustness** and confidence… will still be able to expand trust/explainability of automated mission planning."*

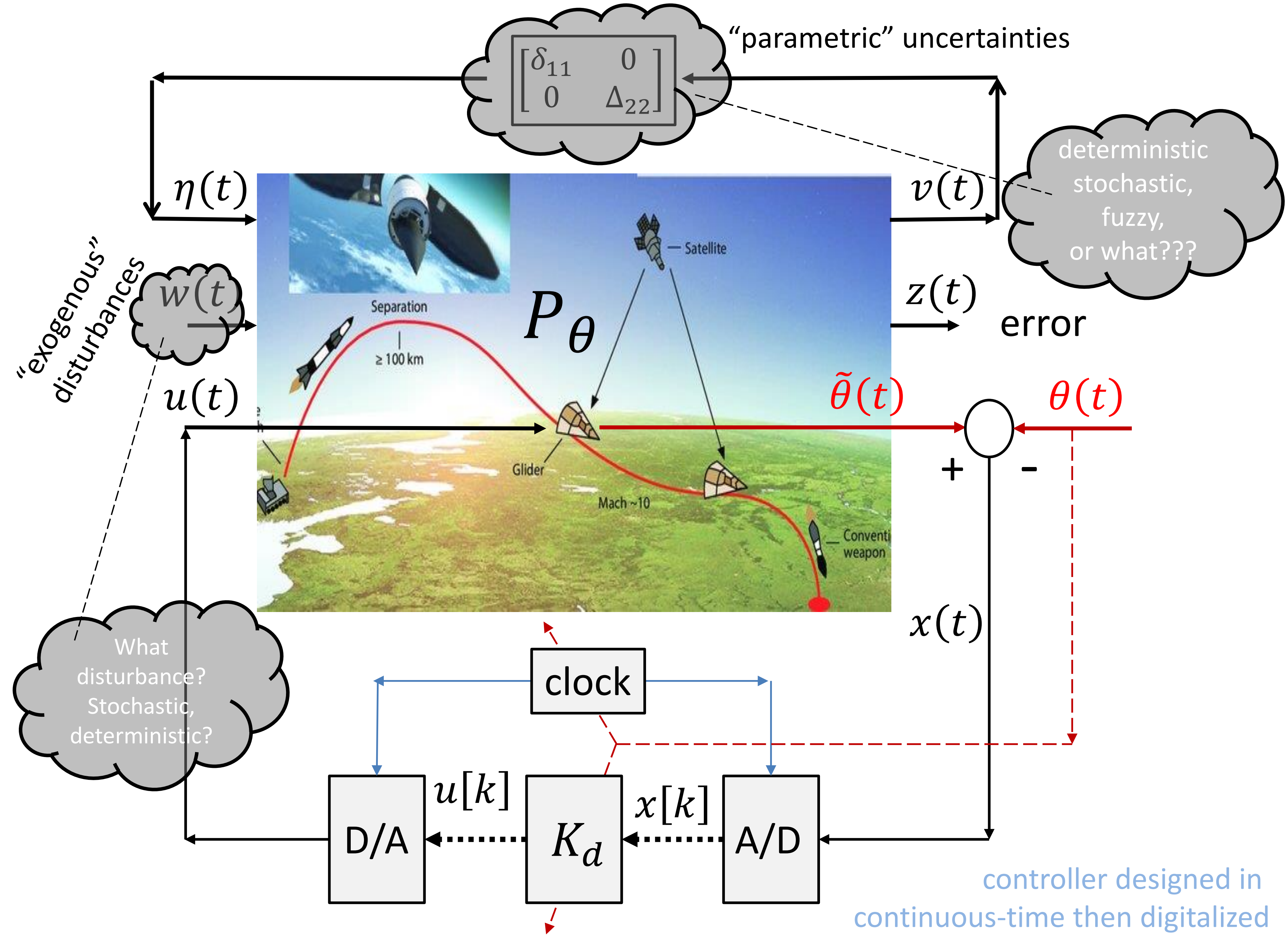


Joint Hypersonics Transition office Announcement TEES/JTHO-RPP-2022-002: Technology Area 4: BUILDING TRUST IN AUTONOMOUS MISSION PLANNING.

Classical Robust Control

- Classical robust control (LQG, H^{∞}) has been extremely successful at designing *uncertainty-aware* control laws
  - when the uncertainties are modeled deterministically.
- The *robust performance theorem* guarantees “hard” error bounds
  - when the uncertainties are subject to “hard” bounds.


"parametric" uncertainties
$\begin{bmatrix} \delta_{11} & 0 \\ 0 & \Delta_{22} \end{bmatrix}$
uncertainty-awareness loop
$\eta(t)$
$v(t)$
"exogenous" disturbances
$w(t)$
$z(t)$
error
P_θ
Satellite
Separation
≥ 100 km
$u(t)$
$\tilde{\theta}(t)$
$\theta(t)$
Preplanned trajectory
+
-
Glider
Mach ~10
Conventi weapon
$x(t)$
clock
$u[k]$
$x[k]$
D/A
K_d
A/D
Warning of massive amount of computations necessitating adaptive sampling



"parametric" uncertainties
$\begin{bmatrix}\delta_{11} & 0\\ 0 & \Delta_{22}\end{bmatrix}$
deterministic
stochastic,
fuzzy,
or what???
$\eta(t)$
$v(t)$
"exogenous" disturbances
$w(t)$
$z(t)$
error
P_θ
Separation
≥ 100 km
Satellite
Glider
Mach ~10
weapon
$u(t)$
$\tilde{\theta}(t)$
$\theta(t)$
+
-
$x(t)$
What disturbance? Stochastic, deterministic?
clock
$u[k]$
$x[k]$
D/A
K_d
A/D
controller designed in continuous-time then digitalized

Critique

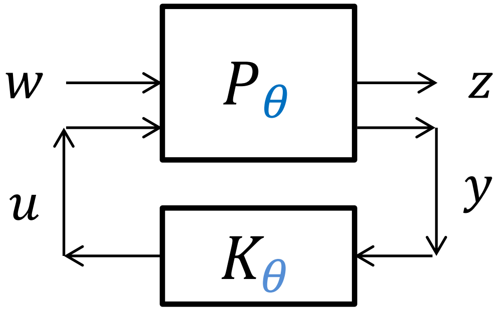
- The *"Achilles heel"* of classical robust control is the modeling of the uncertainty.
- If the modeling of the uncertainty cannot be trusted, the robust control edifice is crumbling.
- Need for trustworthiness assessment
- Quantum control gave us a "heads up."

I. Khalid, C. A. Weidner, E. Jonckheere, S. G. Schirmer, and F. Langbein, ``Statistically characterizing robustness and fidelity of quantum controls and quantum control algorithms," *Physical Review A*, vol. 107, page 032606 (22 pages), March 2023.

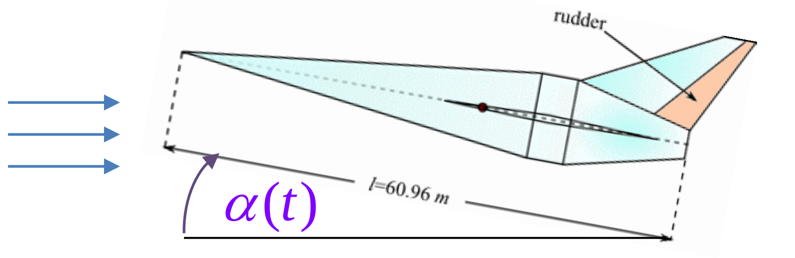
II. S. P. O'Neil, I. Khalid, A. A. Rompokos, C. A. Weidner, F. C. Langbein, S. Shermer, and E. A. Jonckheere, "Analyzing and unifying robustness measures for excitation transfer control in spin networks," *IEEE Control Systems Society Letters,* vol. 7, pp. 1783-1788, 2023

Linear Dynamically Varying ***Uncertainty-Unaware*** Approach

$$P_\theta: \quad \frac{dx(t)}{dt} = A_{\theta(t)}x(t) + B_{\theta(t)}u(t) + F_{\theta(t)}w(t)$$



$$\begin{cases} \frac{dx(t)}{dt} = A_{\theta(t)}x(t) + B_{\theta(t)}K_{\theta(t)}y(t) + F_{\theta(t)}w(t) + G_{\theta(t)}\eta(t) \\ x(t) = \tilde{\theta}(t) - \theta(t) \\ y(t) = C_{\theta(t)}x(t) \\ z(t) = \begin{pmatrix} x(t) \\ \delta\alpha(t) \end{pmatrix} \\ u(t) = K_{\theta(t)}y(t) \end{cases}$$



$$L^1: \min_u \max_w \frac{\max_\tau \|z(\tau)\|}{\max_\tau \|w(\tau)\|}$$

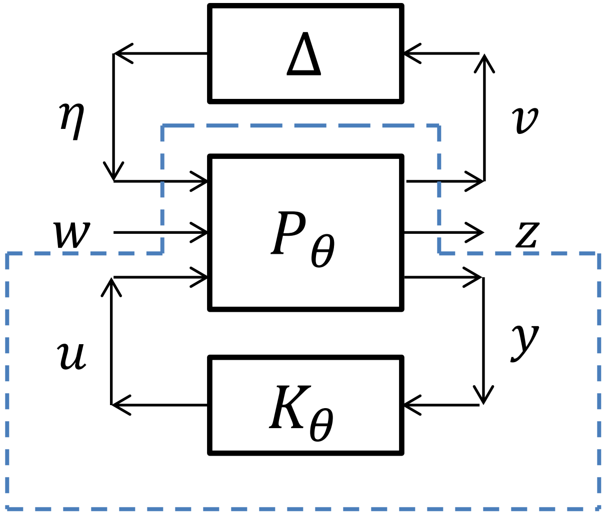
$$H^\infty: \min_u \underbrace{\left(\max_w \frac{\int_t^{t_f} \|z(\tau)\|^2 \, d\tau}{\int_t^{t_f} \|w(\tau)\|^2 \, d\tau} \right)^{1/2}}_{\|T_{z \leftarrow w}\|}$$

S. Bohacek and E. A. Jonckheere, ``Nonlinear tracking over compact sets with Linear Dynamically Varying H^∞ control,'' *SIAM J. Control and Optimization,* vol. 40, No. 4, pp. 1042-1071, 2001.

E. A. Jonckheere, P. Lohsoonthorn, S. Dalzell, ``Eigen-structure versus H^∞ constrained design for hypersonic winged cone,'' *Journal of Guidance, Dynamics and Control,* AIAA, Vol. 24, No., 4, pp. 648-658, July-August 2001.

Linear Dynamically Varying ***Uncertainty-Aware*** Approach

$$P_\theta: \quad \frac{dx(t)}{dt} = A_{\theta(t)}x(t) + B_{\theta(t)}u(t) + F_{\theta(t)}w(t)$$



$F_\ell(P_\theta, K_\theta)$

$$\begin{cases} \dfrac{dx(t)}{dt} = A_{\theta(t)}x(t) + B_{\theta(t)}K_{\theta(t)}y(t) + F_{\theta(t)}w(t) + G_{\theta(t)}\eta(t) \\ x(t) = \tilde{\theta}(t) - \theta(t) \\ y(t) = C_{\theta(t)}x(t) \\ z(t) = \begin{pmatrix} x(t) \\ \delta\alpha(t) \end{pmatrix} \\ u(t) = K_{\theta(t)}y(t) \\ v(t) = D_{\theta(t)}x(t) \\ \eta(t) = \Delta v(t) \end{cases}$$

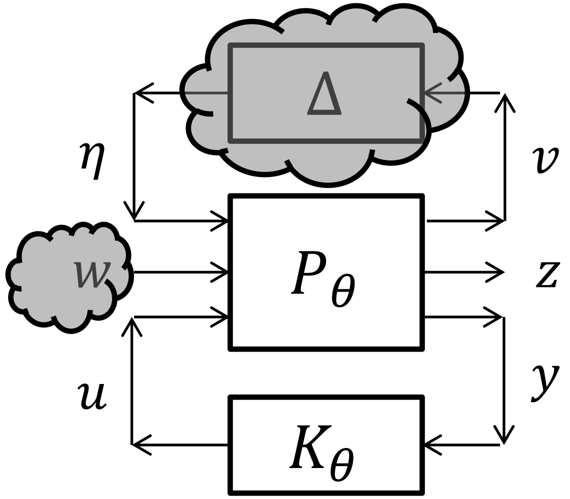
Robust performance theorem:

$$\min_{K_{\theta(t)}} \mu\left(F_\ell\left(P_\theta, K_\theta\right)\right) \Rightarrow \left\| T_{z \leftarrow w}\left(\Delta\right) \right\| \le \mu, \quad \forall \left\|\Delta\right\| < 1/\mu$$

E. A. Jonckheere, P. Lohsoonthorn, and S. K. Bohacek, ``From Sioux City to the X-33,'' (invited paper), *Annual Reviews in Control,* vol. 23, Elsevier, Pergamon, pp. 91-108, 1999.

Linear Dynamically Varying ***Trust-Aware*** Approach

$$P_\theta: \quad \frac{dx(t)}{dt} = A_{\theta(t)}x(t) + B_{\theta(t)}u(t) + F_{\theta(t)}w(t)$$



$$\begin{cases} \frac{dx(t)}{dt} = A_{\theta(t)}x(t) + B_{\theta(t)}K_{\theta(t)}y(t) + F_{\theta(t)}w(t) + G_{\theta(t)}\eta(t) \\ x(t) = \tilde{\theta}(t) - \theta(t) \\ y(t) = C_{\theta(t)}x(t) \\ z(t) = \begin{pmatrix} x(t) \\ \delta\alpha(t) \end{pmatrix} \\ u(t) = K_{\theta(t)}y(t) \\ v(t) = D_{\theta(t)}x(t) \\ \eta(t) = \Delta v(t) \end{cases}$$

Robust performance theorem:

$$\min_{K_{\theta(t)}} \mu\left(F_\ell\left(P_\theta, K_\theta\right)\right) \Rightarrow \left\| T_{z\leftarrow w}(\Delta) \right\| \leq \mu, \quad \forall \left\| \Delta \right\| < 1/\mu$$

How sure are we about this if we are not sure of the uncertainty model?

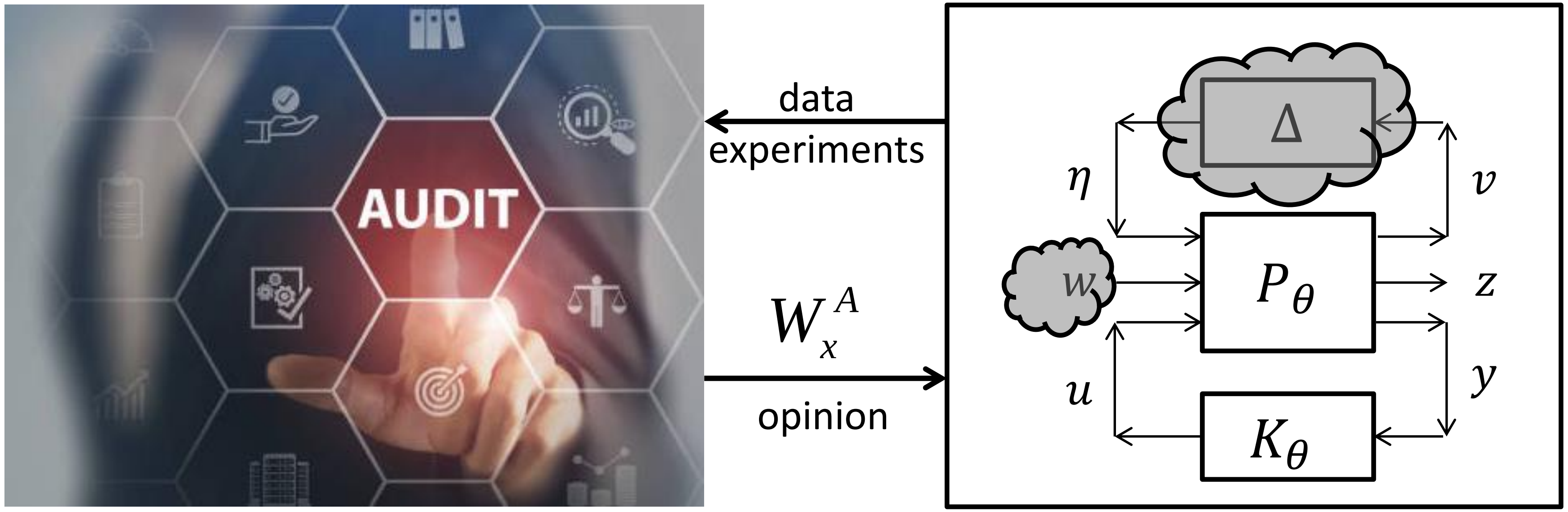
Subjective Logic

uncertain probability = subjective opinion

- We need an analyst or auditor to assess trustworthiness of the design.

Auditor or Analyst, or trustor, A

Designer, or trustee, x



Mingxi Cheng, Shahin Nazarian, and Paul Bogdan, ["There is hope after all: Quantifying opinion and trustworthiness in neural networks,"](#) *Frontiers in Artificial Intelligence*, 3:54, 2020.

Formal Trust Framework

- Positive evidence r_x^A: Trustor *A* find that trustee *x*'s behavior meets some specifications.
- Negative evidence s_x^A: Trustor *A* find that trustee *x*'s behavior does not satisfy specifications.
- Non-informative prior weight *W* default value *W*=2

- Belief $b_x^A = \frac{r_x^A}{r_x^A + s_x^A + W}$
- Disbelief $d_x^A = \frac{s_x^A}{r_x^A + s_x^A + W}$
- Uncertainty $u_x^A = \frac{W}{r_x^A + s_x^A + W}$
- Base rate a_x^A
  - Prior probability without evidence default value $a_x^A = 0.5$

Opinion: $W_x^A = \left(b_x^A, d_x^A, u_x^A, a_x\right)$

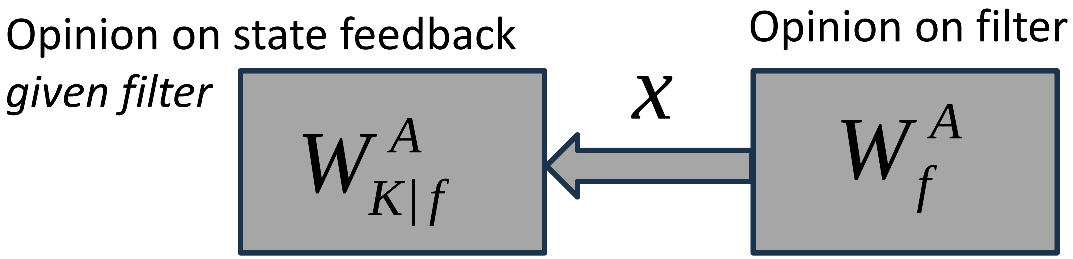
Trustworthiness: $T_x^A = b_x^A + u_x^A a_x^A$

Risk: $R_x^A = d_x^A + u_x^A \left(1 - a_x^A\right)$

Algebra of Opinions

- *Multiplication* of opinions by the same auditor on different sub-designs x, y:

$$W_{x \cdot y} = W_x \cdot W_y : \begin{cases} b_{x \wedge y} = b_x b_y + \dfrac{a_y (1 - a_x) b_x u_y + a_x (1 - a_y) b_y u_x}{1 - a_x a_y} \\ d_{x \wedge y} = d_x + d_y - d_x d_y \\ u_{x \wedge y} = u_x u_y + \dfrac{(1 - a_x) b_y u_x + (1 - a_y) b_x u_y}{1 - a_x a_y} \\ a_{x \wedge y} = a_x a_y \end{cases}$$



$$W^A_K = W^A_{K|f} \cdot W^A_f$$

Algebra of Opinions

- *Fusion* of opinions of two auditors on the same design x,

$$W_x^{A\circ B} = W_x^A \circ W_x^B : \begin{cases} b_x^{A\circ B} = \dfrac{b_x^A u_x^B + b_x^B u_x^A}{u_x^A + u_x^B} \\ u_x^{A\circ B} = \dfrac{2u_x^A u_x^B}{u_x^A + u_x^B} \\ a_x^{A\circ B} = \dfrac{a_x^A + a_x^B}{u_x^A + u_x^B} \end{cases}$$

Trustworthiness of Shapiro (Lockheed) eigenvector assignment

0,1 X

$$r_v^A = 4, s_v^A = 3$$

$$r_\alpha^A = 3, s_\alpha^A = 4$$

$$r_q^A = 1, s_q^A = 6$$

$$r_\vartheta^A = 1, s_\vartheta^A = 6$$

$$r_h^A = 1, s_h^A = 6$$

Table 2 Desired eigenvectors

Parameter	V_p		V_s		V_a	V_e	V_f
Eigenvector	v_1	v_2	v_3	v_4	v_5	v_6	v_7
Velocity	x[a]	1[b]	0[c]	0	0	x	x
Angle of attack	0	0	x	1	x	x	x
Pitch rate	x	x	1	x	x	x	x
Pitch attitude	1	x	x	x	x	x	x
Altitude	x	x	x	x	1	x	x
Symmetric elevon	x	x	x	x	x	1	0
Fuel equivalent ratio	x	x	x	x	x	0	1

[a]Here x is an unspecified component.
[b]Here 1 means that some coupling should be present.
[c]Here 0 means that there should be no coupling.

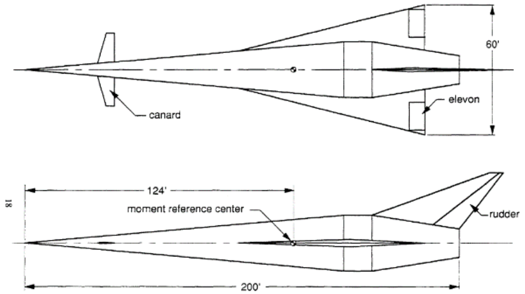
$$T_v^A = \frac{5}{9}, R_v^A = \frac{4}{9}$$

$$T_\alpha^A = \frac{4}{9}, R_\alpha^A = \frac{5}{9}$$

$$T_q^A = \frac{2}{9}, R_q^A = \frac{7}{9}$$

$$T_\vartheta^A = \frac{2}{9}, R_\vartheta^A = \frac{7}{9}$$

$$T_h^A = \frac{2}{9}, R_h^A = \frac{7}{9}$$



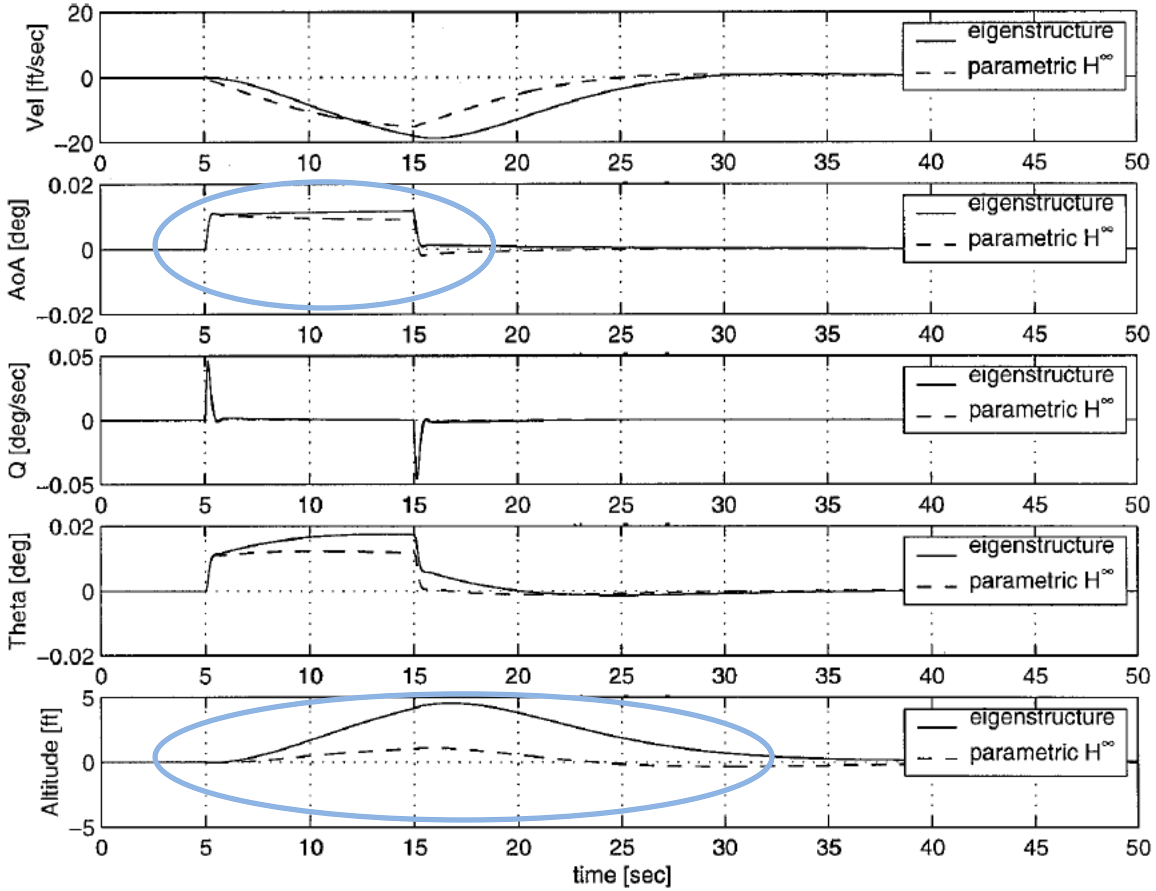
E. Y. Shapiro and J. C. Chung, "Flight control system synthesis using eigenstructure assignment. *J Optim. Theory Appl.*, Vol. 43, pp. 415–429, 1984.

E. A. Jonckheere, P. Lohsoonthorn, S. Dalzell, ``Eigen-structure versus H^∞ constrained design for hypersonic winged cone,'' *Journal of Guidance, Dynamics and Control,* AIAA, Vol. 24, No., 4, pp. 648-658, July-August 2001.

Trustworthiness and Risk consistent with simulation results

$$T_\alpha^A = \frac{4}{9}$$

$$R_\alpha^A = \frac{5}{9}$$



$$T_h^A = \frac{2}{9}$$

$$R_h^A = \frac{7}{9}$$

Fig. 8 Velocity, angle-of-attack, pitch-rate, pitch-angle, and altitude time-domain responses to elevon command.

Trustworthiness higher on angle of attack than altitude

Risk higher on altitude than angle of attack

Off-line trustworthy trajectory planning

Uncertainty-aware planning

- Minimize the *error,* which includes the targeting error

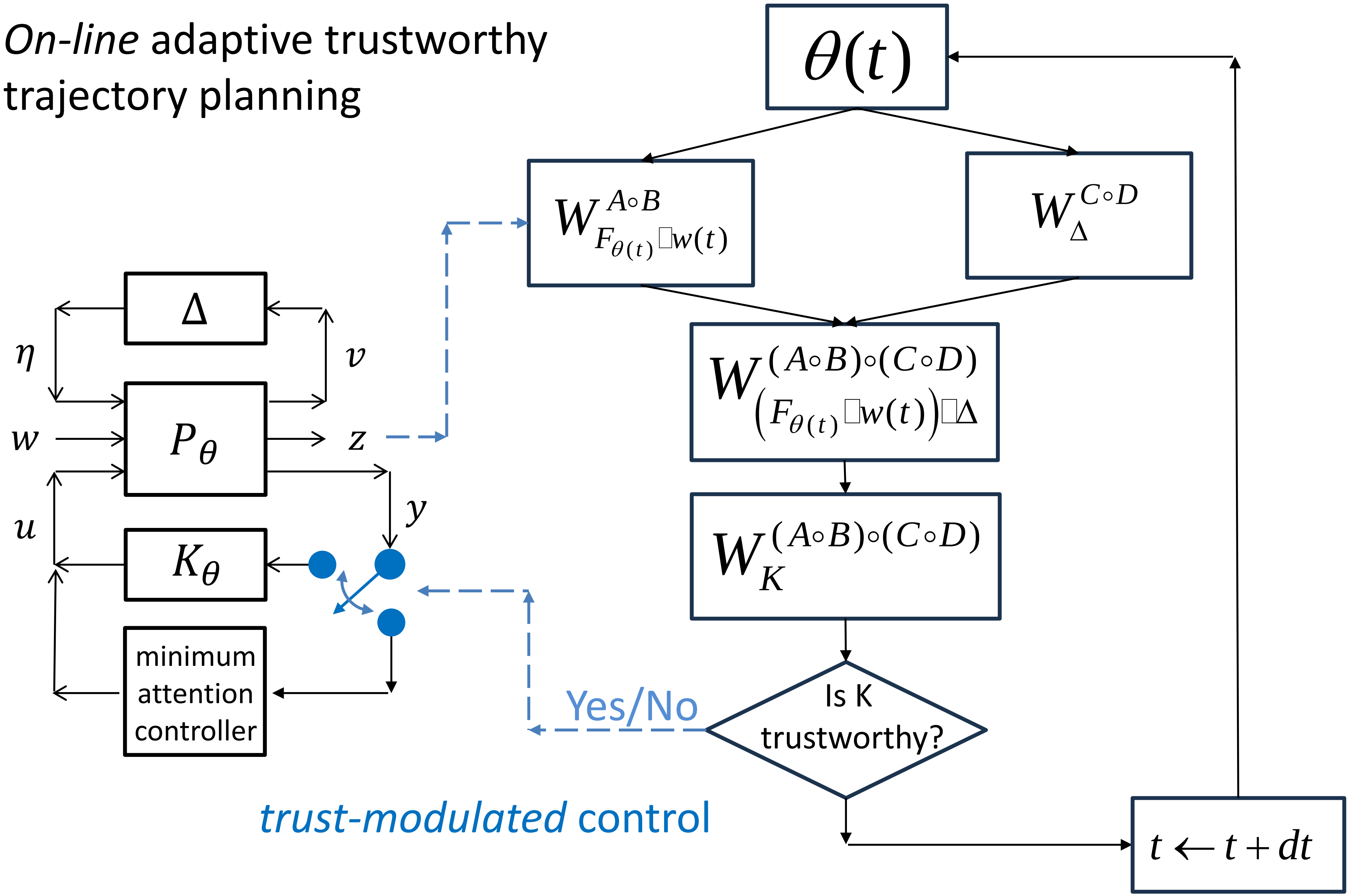
$$\min_{\Theta}\left(\min_{K_\theta} \mu\left(F_\ell\left(P_\theta, K_\theta\right)\right)\right)$$

Trust-aware planning

- Minimize the *risk* of missing the target

$$\min_{\Theta} R_{K_\theta}^{(A\circ B)\circ(C\circ D)}$$

On-line adaptive trustworthy trajectory planning



trust-modulated control

J. Shi and D. W. Appley, "A suboptimal N-Step-Ahead cautious controller for adaptive control applications," *J. Dynamic Systems, Measurements and Control,* vol. 120, pp. 419-423, Sept. 1998.
R. W. Brockett, "Minimum attention control," *Proceedings of the 36th IEEE Conference on Decision and Control,"* San Diego, CA, December 1997, pp. 2628, 1997.

Conclusions

- Hypersonic mission planning must take into consideration poorly known uncertainties.
- Classical robust control has failed to address trustworthiness of the modeling of the uncertainties.
- We proposed both *off-line* and *on-line* trustworthiness assessments of hypersonic glide vehicles trajectory planning based on subjective logic.
- Early results on a NASA demonstration vehicle showed the viability of the approach.

Thank you!

Questions?

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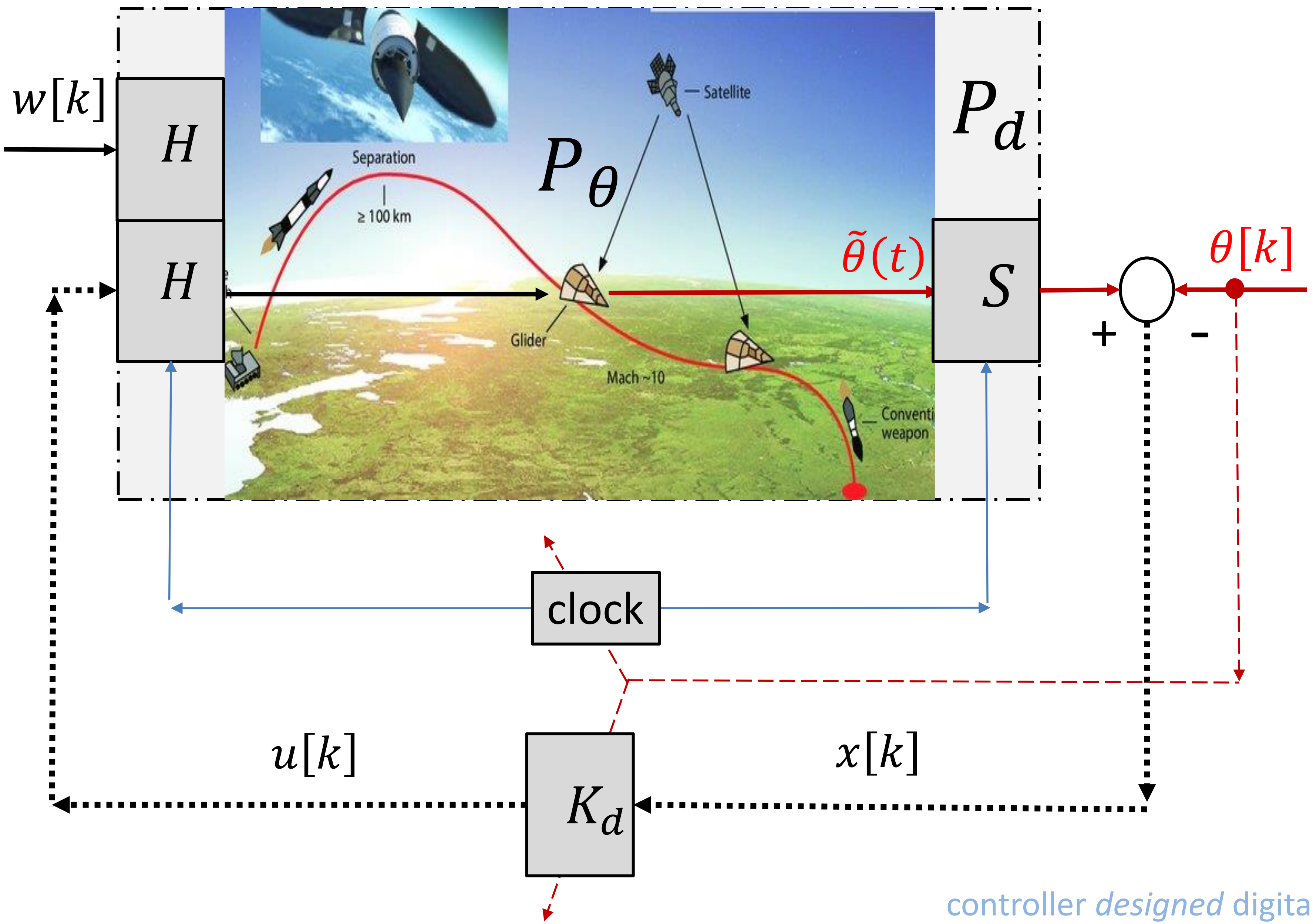
Last warning: Massive amount of computations probably requiring variable sampling rate sampled data adaptive control



controller *designed* digitally

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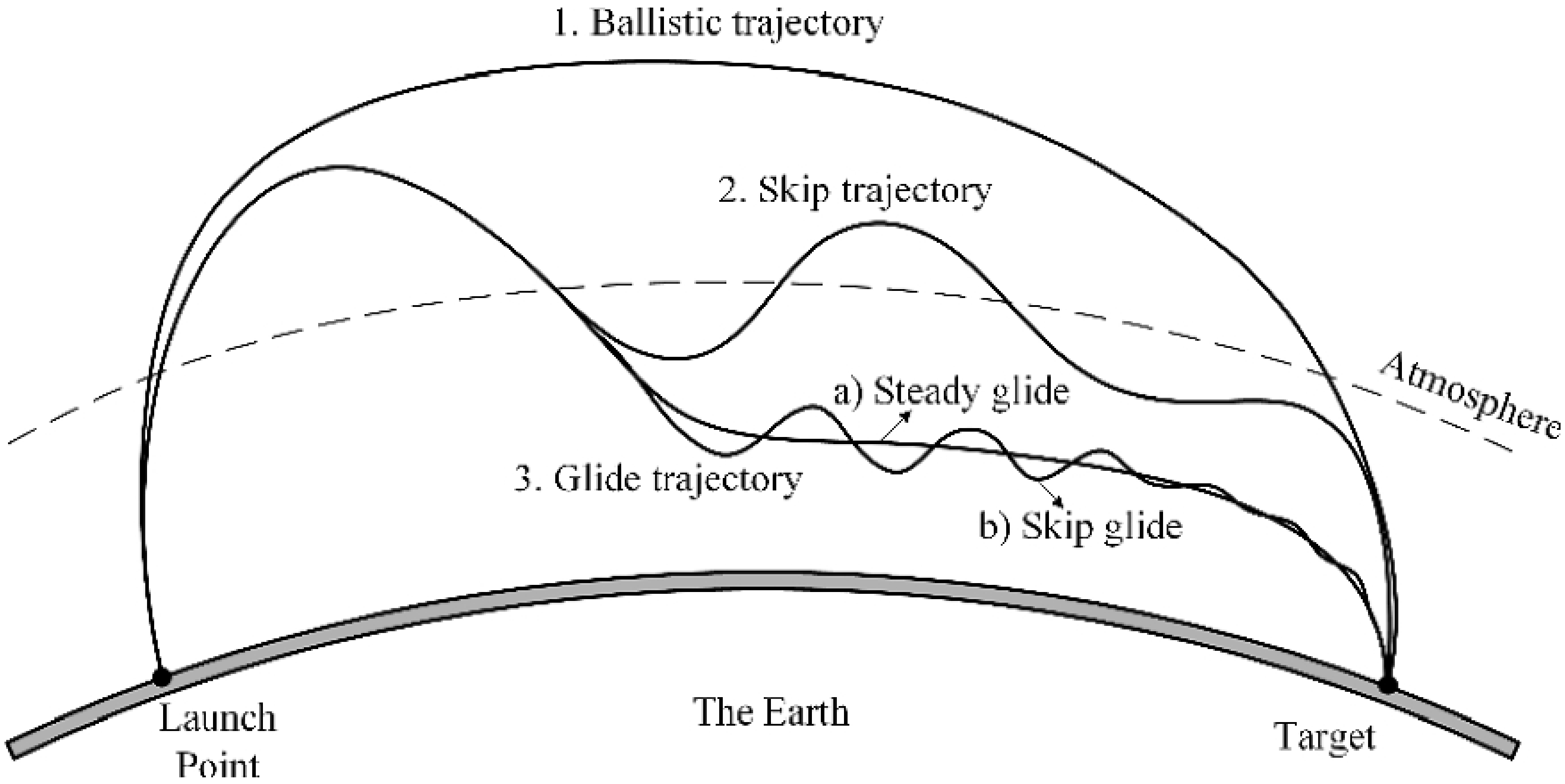

Joint Hypersonics Transition office Announcement TEES/JTHO-RPP-2022-002: Technology Area 4: BUILDING TRUST IN AUTONOMOUS MISSION PLANNING.

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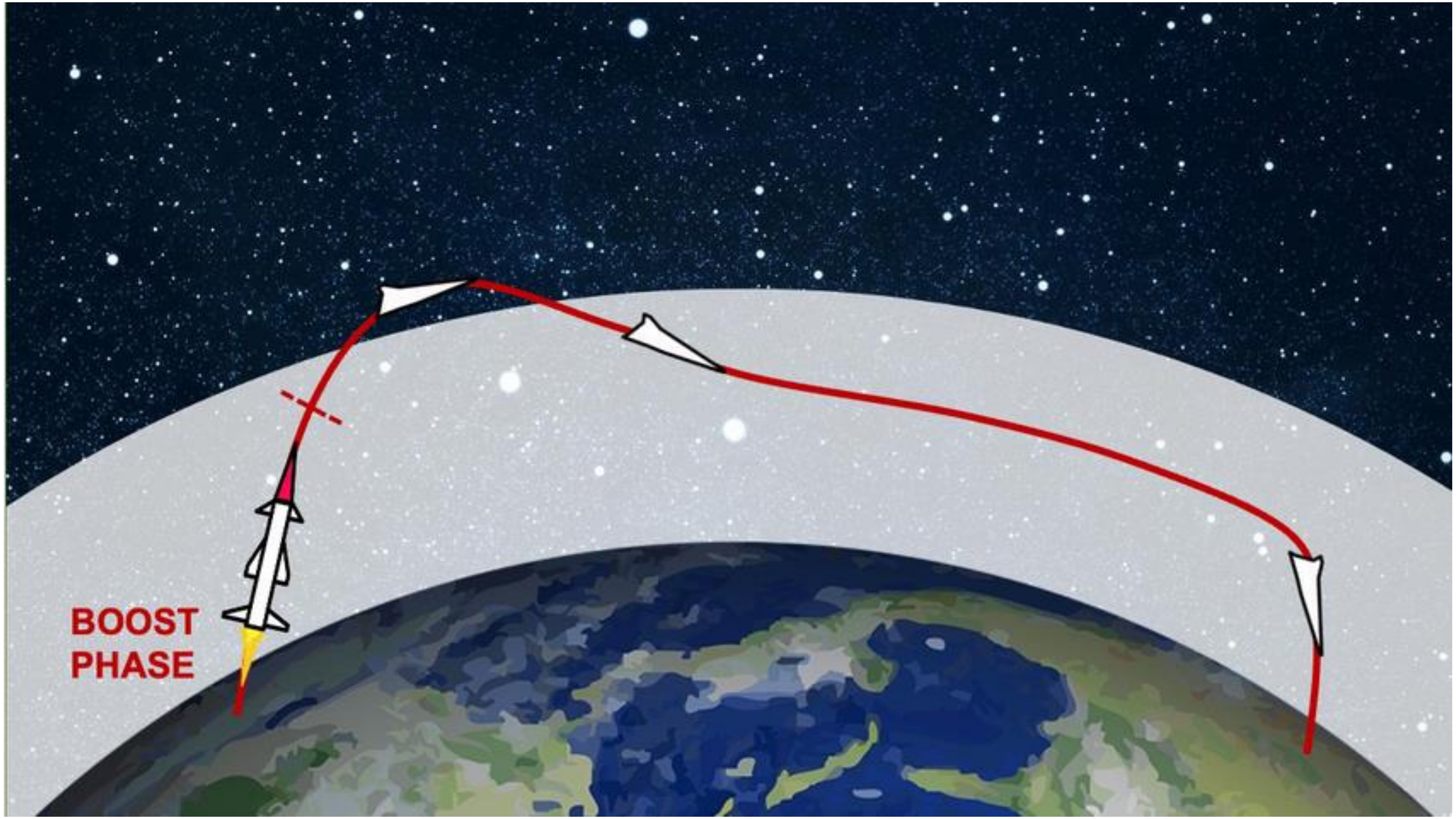

Joint Hypersonics Transition Office Announcement TEES/JTHO-RPP-2022-002: Technology Area 4: BUILDING TRUST IN AUTONOMOUS MISSION PLANNING.

$$
\begin{cases}
\dfrac{dx(t)}{dt} = A_{\theta(t)}x(t) + B_{\theta(t)}L_{\theta(t)}y_{\theta}(t) + F_{\theta(t)}w(t) + G_{\theta(t)}\eta(t) \\
x = \tilde{\theta} - \theta \\
y(t) = C_{\theta(t)}x(t) \\
z(t) = \begin{cases} x(t) \\ \text{couplings} \end{cases} \\
v(t) = D_{\theta(t)}x(t) \\
\eta(t) = \Delta v(t) \\
u(t) = K_{\theta(t)}y(t)
\end{cases}
$$

$$
\begin{cases}
\dfrac{dx(t)}{dt} = A_{\theta(t)} + B_{\theta(t)}L_{\theta(t)}y(t) + F_{\theta(t)}w(t) + G_{\theta(t)}\eta(t) \\
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z(t) = \begin{pmatrix} x(t) \\ \text{aero-coupling} \end{pmatrix} \\
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$$W_{x\wedge y}\begin{cases} b_{x\wedge y}=b_x b_y+\dfrac{(1-a_x)a_y b_x u_y+a_x(1-a_y)b_y u_x}{1-a_x a_y} \\ d_{x\wedge y}=d_x+d_y-d_x d_y \\ u_{x\wedge y}=u_x u_y+\dfrac{(1-a_x)b_y u_x+(1-a_y)b_x u_y}{1-a_x a_y} \\ a_{x\wedge y}=a_x a_y \end{cases}$$

$$W_x^{A\circ B}\begin{cases} b_x^{A\circ B} = \dfrac{b_x^A u_x^B + b_x^B u_x^A}{u_x^A + u_x^B} \\ u_x^{A\circ B} = \dfrac{2u_x^A u_x^B}{u_x^A + u_x^B} \\ a_x^{A\circ B} = \dfrac{a_x^A + a_x^B}{u_x^A + u_x^B} \end{cases}$$

Upon examination of a design x (e.g., a hypersonic mission planning), the Trustor could bring the following evidence:

- Positive evidence that the mission achieves some objectives, as quantified by a score $r(A, x)$,
- Negative evidence that the mission falls short of some objectives, quantified by a score $d(A, x)$,
- Lack of prior evidence, quantify by a weight W.

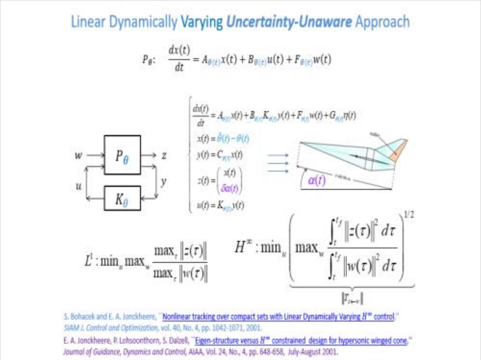
The scores $r(A, x)$ and $d(A, x)$ could be the number of times a digital twin achieves, resp. fails to achieve, the mission objectives. The weight W could be the number of times the repeated experiments provide neither positive nor negative evidence that the mission objectives are achieved.

Given such quantification of evidence, the next step is normalization of the scores:

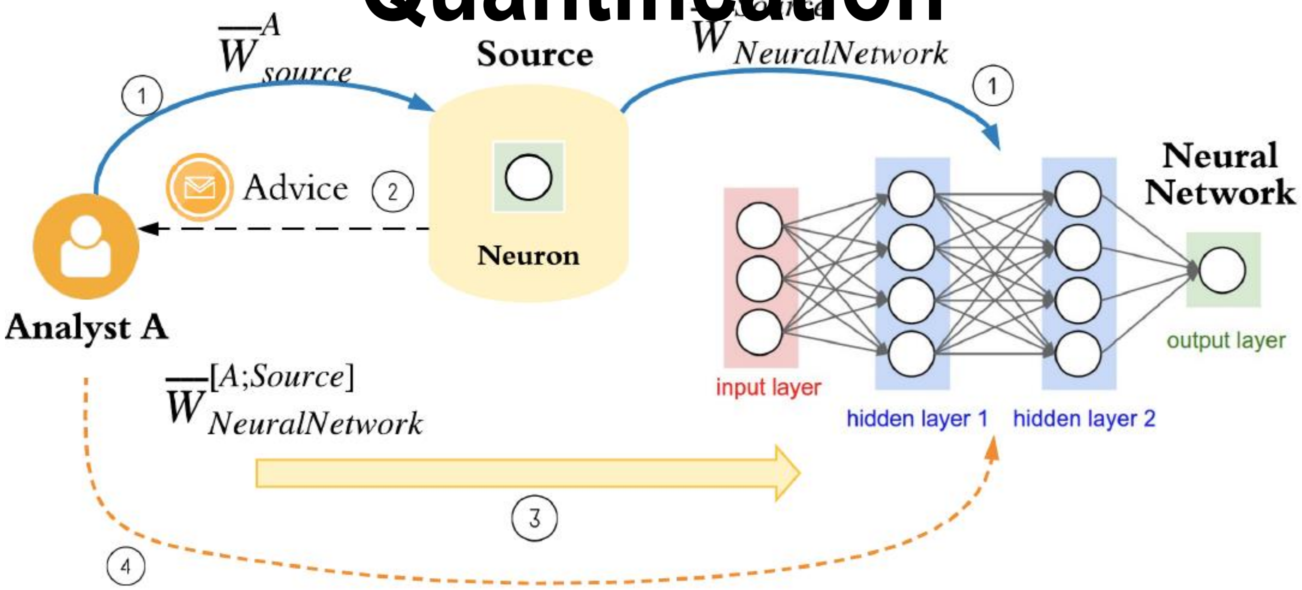
- Belief, quantified by $b(A, x) = \frac{r(A,X)}{r(A,x)+d(A,x)+W} \in [0,1]$
- Disbelief, quantified by $d(A, x) = \frac{d(A,X)}{r(A,x)+d(A,x)+W} \in [0,1]$
- Uncertainty, quantified by $u(A, x) = \frac{W}{r(A,x)+d(A,x)+W} \in [0,1]$

So far, the discourse is probabilistic in so far as belief, disbelief, and uncertainty can be interpreted as frequency of reoccurrence of positive, negative, or no evidence.

- **Trustworthiness:** $p = belief + ignorance * base\_rate$
- **Risk:** $k = disbelief + ignorance * (1 - base\_rate)$



Recap: DeepTrust - DNN Trust Quantification



$$\overline{W}_{NN} = \overline{W}^{A}_{NeuralNetwork} = fusion(\overline{W}^{[A;Source]}_{NeuralNetwork})$$

- **Quantify the trustworthiness of a DNN requires:**
  - Subjective trust network formulation
  - Trustworthiness of dataset
  - Architecture of the neural network

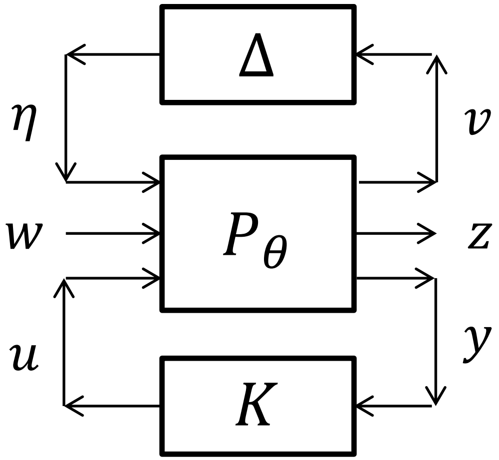
More details

Cheng Mingxi, Shahin Nazarian, and Paul Bogdan, “There Is Hope After All: Quantifying Opinion and Trustworthiness of Neural Networks”, Frontiers in Artificial Intelligence, 2020

Linear Dynamically Varying (LDV) Approach

$$P_\theta: \quad \frac{dx(t)}{dt} = A_{\theta(t)}x(t) + B_{\theta(t)}u(t) + F_{\theta(t)}w(t)$$

$$F(Q_\theta, L_\theta)\begin{cases} \dfrac{dx(t)}{dt} = A_{\theta(t)}x(t) + B_{\theta(t)}L_{\theta(t)}y_\theta(t) + F_{\theta(t)}w(t) + G_{\theta(t)}\eta(t) \\ \tilde{\theta} = x + \theta \\ y_\theta(t) = C_{\theta(t)}x(t) \\ z(t) = \begin{cases} x(t) \\ couplings \end{cases} \\ \upsilon(t) = D_{\theta(t)}x(t) \end{cases}$$

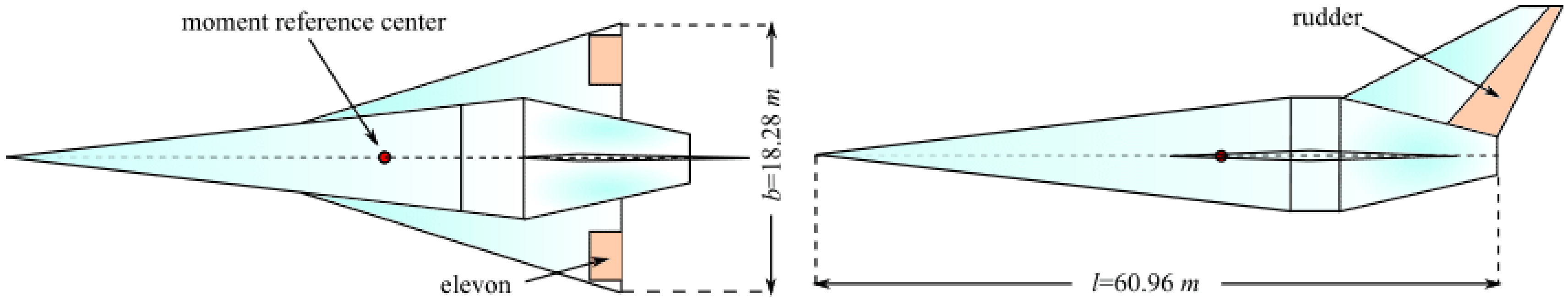


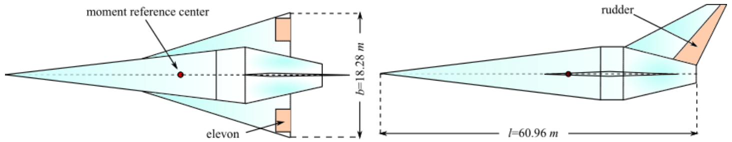
$$H^\infty: \; min_u max_w \frac{\int_t^{t_f}\big(x'(\tau)Qx(\tau) + u'(\tau)Ru(\tau)\big)d\tau}{\int_t^{t_f}(\|w(\tau)\|^2 d\tau)}$$

$$L^1: \; min_u max_{\tau,w} \frac{x'(\tau)Qx(\tau) + u'(\tau)Ru(\tau)}{\|w(\tau)\|^2}$$


AUDIT

INTERNAL
AUDIT


moment reference center
elevon
b=18.28 m
rudder
l=60.96 m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b=18.28 m
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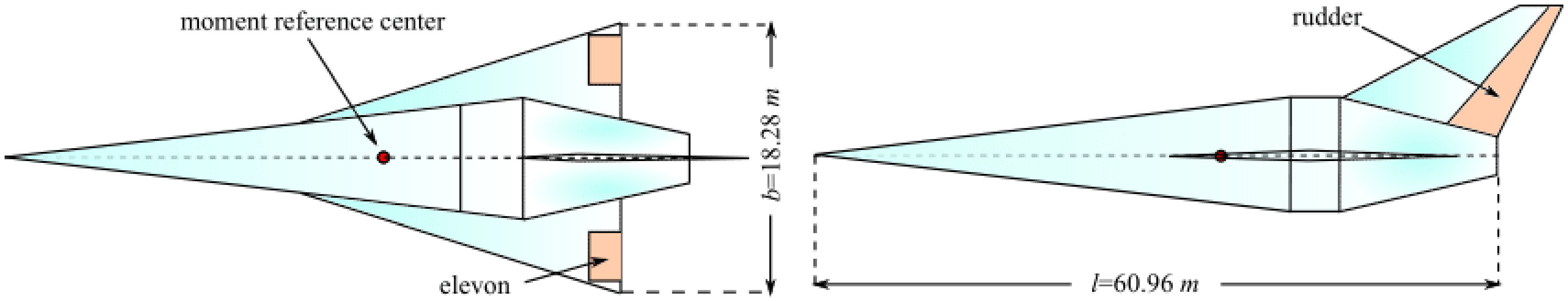
moment reference center
elevon
b=18.28 m
rudder
l=60.96 m

$$\begin{bmatrix} \delta_{11} & 0 \\ 0 & \Delta_{22} \end{bmatrix}$$